Massachusetts Institute of Technology

Department of Electrical Engineering and Computer Science

6.111 - Introductory Digital Systems Laboratory

Problem Set 1

Issued: September 3, 2003 **Due:** September 10, 2003

Problem 1: Boolean Algebra Practice Problems *(Problem 1 will not be graded.)*
Simplify each expression by algebraic manipulation. Try to recognize when it is appropriate to transform to the dual, simplify, and re-transform (e.g. no. 6). Try doing the problems before looking at the solutions which are at the end of this problem set.

1) $a + 0 =$
2) $\overline{a}.0 =$
3) $a + \overline{a} =$
4) $a + a =$
5) $a + ab =$
6) $a + \overline{a}b =$
7) $a(\overline{a} + b) =$
8) $ab + \overline{a}b =$
9) $(\overline{a} + \overline{b})(\overline{a} + b) =$
10) $a(a + b + c + ...) =$

For (11),(12), (13), $f(a, b, c) = a + b + c$

11) $f(a, b, ab) =$
12) $f(a, b, \overline{a}\overline{b}) =$
13) $f[a, b, \overline{(ab)}] =$
14) $y + y\overline{y} =$
15) $xy + x\overline{y} =$
16) $\overline{x} + y\overline{x} =$
17) $(w + \overline{x} + y + \overline{z})y =$
18) $(x + \overline{y})(x + y) =$
19) $w + [w + (wx)] =$
20) $x[x + (xy)] =$
21) $\overline{(\overline{x} + \overline{x})} =$
22) $\overline{(x + \overline{x})} =$
23) $w + (w\overline{x}yz) =$
24) $\overline{w}\overline{(wxyz)} =$
25) $xz + \overline{x}y + zy =$
26) $(x + z)(\overline{x} + y)(z + y) =$
27) $\overline{x} + \overline{y} + xy\overline{z} =$

Problem 2:

For each of the following Boolean expressions, give:

i) The truth table,
ii) The Karnaugh map,
iii) The MSP expression, (Show groupings)
iv) The MPS expression. (Show groupings)

1. $(\overline{a} + b \cdot c) + b \cdot (\overline{a} \cdot \overline{c} \cdot \overline{d} + a \cdot c \cdot d)$
2. $(\overline{d} + b \cdot \overline{c}) \cdot (\overline{d} + (\overline{a} + b) \cdot (\overline{c} + d)) \cdot (a + \overline{c})$
3. $(w + \overline{y}) \cdot (\overline{w} + \overline{y} + z) \cdot (w + x + \overline{z})$

Problem 3:

Karnaugh Maps are useful for finding minimal implementations of Boolean expressions with only a few variables. However, they can be a little tricky when "don't cares" (X) are involved. Using the following K-Maps:

1. Find the minimal sum of products expression. Show your groupings.
2. Find the minimal product of sums expression. Show your groupings.
3. Are your solutions unique? If not, list and show the other minimal expressions.
4. Does the MPS = MSP?

cd \ ab	00	01	11	10
00	0	1	X	0
01	0	0	1	0
11	0	0	1	0
10	0	1	1	1

cd \ ab	00	01	11	10
00	1	1	0	1
01	X	0	0	X
11	0	0	0	0
10	0	0	0	0

Problem 4:

Use DeMorgan's Theorems to simplify the following expressions:

1. $\overline{\overline{(\overline{c}+d)}\cdot\overline{(\overline{a}+\overline{c})}}$
2. $\overline{a\cdot c\cdot d}$
3. $\overline{a+c}\cdot\overline{\overline{c}+\overline{d}}\cdot\overline{a+d}$

Solutions to Boolean Algebra Practice Problems

1) $a + 0 = a$
2) $\overline{a}.0 = 0$
3) $a + \overline{a} = 1$
4) $a + a = a$
5) $a + ab = a(1 + b) = a$
6) $a + \overline{a}b = (a + \overline{a})(a + b) = a + b$
7) $a(\overline{a} + b) = a\overline{a} + ab = ab$
8) $ab + \overline{a}b = b(a + \overline{a}) = b$
9) $(\overline{a} + \overline{b})(\overline{a} + b) = \overline{a}\overline{a} + \overline{a}b + \overline{b}\overline{a} + \overline{b}b = \overline{a} + \overline{a}b + \overline{a}\overline{b} = \overline{a}(1 + b + \overline{b}) = \overline{a}$
10) $a(a + b + c...) = aa + ab + ac + ... = a + ab + ac + ... = a$
11) $f(a, b, ab) = a + b + ab = a + b$
12) $f(a, b, \overline{a}\overline{b}) = a + b + \overline{a}\overline{b} = a + b + \overline{a} = 1$
13) $f(a, b, \overline{(ab)}) = a + b + \overline{(ab)} = a + b + \overline{a} + \overline{b} = 1$
14) $y + y\overline{y} = y$
15) $xy + x\overline{y} = x(y + \overline{y}) = x$
16) $\overline{x} + y\overline{x} = \overline{x}(1 + y) = \overline{x}$
17) $(w + \overline{x} + y + \overline{z})y = y$
18) $(x + \overline{y})(x + y) = x$
19) $w + (w + (wx)) = w$
20) $\overline{x(x + (xy))} = x$
21) $\overline{(\overline{x} + \overline{x})} = x$
22) $\overline{(x + \overline{x})} = 0$
23) $w + (w\overline{x}yz) = w(1 + \overline{x}yz) = w$
24) $\overline{w}\overline{(wxyz)} = \overline{w}(\overline{w} + \overline{x} + \overline{y} + \overline{z}) = \overline{w}$
25) $xz + \overline{x}y + zy = xz + \overline{x}y$
26) $(x + z)(\overline{x} + y)(z + y) = (x + z)(\overline{x} + y)$
27) $\overline{x} + \overline{y} + xy\overline{z} = \overline{x} + \overline{y} + \overline{z}$