

Massachusetts Institute of Technology  
 Department of Electrical Engineering and Computer Science  
 6.111 - Introductory Digital Systems Laboratory

**Problem Set 1**

**Issued:** September 8, 1999

**Due:** September 15, 1999

**Problem 1: Boolean Algebra Practice Problems**

Simplify each expression by algebraic manipulation. Try to recognize when it is appropriate to transform to the dual, simplify, and retransform (e.g. no. 6). Try doing the problems before looking at the solutions which are at the end of this problem set.

- |  |  |
|--|--|
| 1) $a + 0$                                 | 14) $y + y\bar{y}$                                       |
| 2) $\bar{a} \cdot 0$                       | 15) $xy + x\bar{y}$                                      |
| 3) $a + \bar{a}$                           | 16) $\bar{x} + y\bar{x}$                                 |
| 4) $a + a$                                 | 17) $(w + \bar{x} + y + \bar{z})y$                       |
| 5) $a + ab$                                | 18) $(x + \bar{y})(x + y)$                               |
| 6) $a + \bar{a}b$                          | 19) $w + [w + (wx)]$                                     |
| 7) $a(\bar{a} + b)$                        | 20) $x[x + (xy)]$  |
| 8) $ab + \bar{a}b$                         | 21) $\frac{\overline{\bar{x} + \bar{x}}}{(x + \bar{x})}$ |
| 9) $(\bar{a} + \bar{b})(\bar{a} + b)$      | 22) $\frac{\overline{\bar{x} + \bar{x}}}{(x + \bar{x})}$ |
| 10) $a(a + b + c + \dots)$                 | 23) $w + (w\bar{x}yz)$                                   |
| For (11),(12), (13), $f(a, b, c)a + b + c$ | 24) $\overline{w(wxyz)}$                                 |
| 11) $f(a, b, ab)$                          | 25) $xz + \bar{x}y + zy$                                 |
| 12) $f(a, b, \bar{a}\bar{b})$              | 26) $(x + z)(\bar{x} + y)(z + y)$                        |
| 13) $f[a, b, \overline{(ab)}]$             | 27) $\bar{x} + \bar{y} + xy\bar{z}$                      |

**Problem 2:**

For each of the following Boolean expressions, give:

- i) The truth table,
- ii) The Karnaugh map,
- iii) The MSP expression, (Show groupings)
- iv) The MPS expression. (Show groupings)

1.  $b \cdot (a \cdot \bar{c}) + a \cdot (b \cdot c \cdot \bar{d} + \bar{b} \cdot \bar{c} \cdot d)$
2.  $(a + b \cdot c) \cdot (a + (b + d) \cdot (c + d)) \cdot (\bar{a} + (b + \bar{c}) \cdot (c + d))$
3.  $w \cdot x + w \cdot \bar{x} \cdot \bar{y} + \bar{w} \cdot x \cdot y$

**Problem 3:**

Karnaugh Maps are useful for finding minimal implementations of Boolean expressions with only a few variables. However, they can be a little tricky when “don’t cares” (X) are involved. Using the following K-Maps:

1. Find the minimal sum of products expression. Show your groupings.
2. Find the minimal product of sums expression. Show your groupings.
3. Are your solutions unique? If not, list and show the other minimal expressions.
4. Does the MPS = MSP?

		WX			
	YZ	00	01	11	10
00		1	1	1	1
01		0	1	X	1
11		X	0	1	1
10		0	0	0	0

		AB			
	CD	00	01	11	10
00		1	1	X	1
01		0	0	0	0
11		0	0	X	1
10		1	0	1	X

**Problem 4:**

Use Demorgan’s Theorems to simplify the following expressions:

1.  $\overline{(\bar{a} + \bar{b})} \cdot (a + \bar{c})$
2.  $\overline{\bar{a} \cdot \bar{b} \cdot c}$
3.  $\overline{(\bar{b} \cdot \bar{c})} \cdot \overline{(\bar{a} \cdot \bar{b})} \cdot \overline{(\bar{a} \cdot c)}$

**Problem 5:**

Do the mouse-clicking, etc., as specified by the file

`/mit/6.111/vhdl/warp/beginner/beginners.guide`

Create a VHDL file, `p5.vhd`, to implement

$$p = b \cdot (a \cdot \bar{c}) + a \cdot (b \cdot c \cdot \bar{d} + \bar{b} \cdot \bar{c} \cdot d) \quad (\text{problem 2-1})$$

and

$$q = \overline{(\bar{a} + \bar{b})} \cdot (a + \bar{c}) \quad (\text{problem 4-1})$$

Use the sample file, `/mit/6.111/vhdl/warp/beginner/beginners.guide/halfadd.vhd`, as a guide. Process this VHDL file, choosing a 20V8 as the device.

Verify that the “correct” answer is generated by inspecting the report file.

## Solutions to Boolean Algebra Practice Problems

- 1)  $a + 0 = a$
- 2)  $\bar{a} \cdot 0 = 0$
- 3)  $a + \bar{a} = 1$
- 4)  $a + a = a$
- 5)  $a + ab = a(1 + b) = a$
- 6)  $a + \bar{a}b = (a + \bar{a})(a + b) = a + b$
- 7)  $a(\bar{a} + b) = a\bar{a} + ab = ab$
- 8)  $ab + \bar{a}b = b(a + \bar{a}) = b$
- 9)  $(\bar{a} + \bar{b})(\bar{a} + b) = \bar{a}\bar{a} + \bar{a}b + \bar{b}\bar{a} + \bar{b}b = \bar{a} + \bar{a}b + \bar{a}\bar{b} = \bar{a}(1 + b + \bar{b}) = \bar{a}$
- 10)  $a(a + b + c\dots) = aa + ab + ac + \dots = a + ab + ac + \dots = a$
- 11)  $f(a, b, ab) = a + b + ab = a + b$
- 12)  $f(a, b, \bar{a}\bar{b}) = a + b + \bar{a}\bar{b} = a + b + \bar{a} = 1$
- 13)  $f(a, b, \overline{(ab)}) = a + b + \overline{(ab)} = a + b + \bar{a} + \bar{b} = 1$
- 14)  $y + y\bar{y} = y$
- 15)  $xy + x\bar{y} = x(y + \bar{y}) = x$
- 16)  $\bar{x} + y\bar{x} = \bar{x}(1 + y) = \bar{x}$
- 17)  $(w + \bar{x} + y + \bar{z})y = y$
- 18)  $(x + \bar{y})(x + y) = x$
- 19)  $w + (w + (wx)) = w$
- 20)  $x(x + (xy)) = x$
- 21)  $\overline{(\bar{x} + \bar{x})} = x$
- 22)  $\overline{(x + \bar{x})} = 0$
- 23)  $w + (w\bar{x}yz) = w(1 + \bar{x}yz) = w$
- 24)  $\bar{w}(wxyz) = \bar{w}(\bar{w} + \bar{x} + \bar{y} + \bar{z}) = \bar{w}$
- 25)  $xz + \bar{x}y + zy = xz + \bar{x}y$
- 26)  $(x + z)(\bar{x} + y)(z + y) = (x + z)(\bar{x} + y)$
- 27)  $\bar{x} + \bar{y} + xy\bar{z} = \bar{x} + \bar{y} + \bar{z}$