

6.111 Problem Set 1 Solutions

Problem 1

Not Graded.

Problem 2

1. $F = a\bar{b} + b(\bar{a} + c)$

i. Truth Table

| a | b | c | $a\bar{b}$ | $b(\bar{a}+c)$ | F |
|---|---|---|------------|----------------|---|
| 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 0 | 0 | 0 |
| 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 1 | 1 |

ii. K-map

| | | | | | |
|---|---|--------|--------|--------|--------|
| | | A B | | | |
| | | 00 | 01 | 11 | 10 |
| C | 0 | 0 0 | 1 2 | 0 6 | 1 4 |
| | 1 | 0 1 | 1 3 | 1 7 | 1 5 |

| | | | | | |
|---|---|--------|--------|--------|--------|
| | | A B | | | |
| | | 00 | 01 | 11 | 10 |
| C | 0 | 0 0 | 1 2 | 0 6 | 1 4 |
| | 1 | 0 1 | 1 3 | 1 7 | 1 5 |

iii. MSP

$$F = \bar{a}b + a\bar{b} + ac$$

iv. MPS

$$F = (a + b) * (\bar{a} + \bar{b} + c)$$

$$2. F = (bc + d) * (\bar{a} + bc) * (\bar{a} + d)$$

i. Truth Table

| a | b | c | d | bc+d | $\bar{a}+bc$ | $\bar{a}+d$ | F |
|---|---|---|---|------|--------------|-------------|---|
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 |
| 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 0 | 1 | 1 | 1 | 1 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 0 | 1 | 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 1 | 0 | 1 | 0 |
| 1 | 1 | 1 | 0 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 |

ii. K-maps

| | | A B | | | |
|-----|----|--------|--------|---------|---------|
| | | 00 | 01 | 11 | 10 |
| C D | 00 | 0 0 | 4 0 | 12 0 | 8 0 |
| | 01 | 1 1 | 5 1 | 13 0 | 9 0 |
| | 11 | 3 1 | 7 1 | 15 1 | 11 0 |
| | 10 | 2 0 | 6 1 | 14 0 | 10 0 |

| | | A B | | | |
|-----|----|--------|--------|---------|---------|
| | | 00 | 01 | 11 | 10 |
| C D | 00 | 0 0 | 4 0 | 12 0 | 8 0 |
| | 01 | 1 1 | 5 1 | 13 0 | 9 0 |
| | 11 | 3 1 | 7 1 | 15 1 | 11 0 |
| | 10 | 2 0 | 6 1 | 14 0 | 10 0 |

iii. MSP

$$F = \bar{a}d + bcd + \bar{a}bc$$

iv. MPS

$$F = (b + d)(c + d)(\bar{a} + c)(\bar{a} + b)(d + \bar{a})$$

Problem 3

1.

| | | | | | | | |
|-----|----|-----|----|----|----|--|--|
| | | A B | | | | | |
| | | 00 | 01 | 11 | 10 | | |
| C D | 00 | 1 | 1 | 0 | 1 | | |
| | 01 | 1 | X | 0 | 0 | | |
| | 11 | 0 | 0 | 0 | 0 | | |
| | 10 | 1 | 0 | 0 | 1 | | |

$$\text{MSP} = \bar{x} * \bar{z} + \bar{w} * \bar{y}$$

2.

| | | | | | | | |
|-----|----|-----|----|----|----|--|--|
| | | A B | | | | | |
| | | 00 | 01 | 11 | 10 | | |
| C D | 00 | 1 | 1 | 0 | 1 | | |
| | 01 | 1 | X | 0 | 0 | | |
| | 11 | 0 | 0 | 0 | 0 | | |
| | 10 | 1 | 0 | 0 | 1 | | |

$$\text{MPS} = (\bar{w} + \bar{x})(\bar{w} + \bar{z})(\bar{y} + \bar{z})(\bar{y} + \bar{x})$$

3. Both the MPS and the MSP are unique.

4. MSP = MPS, because the don't care is treated as a '1' in both cases

1.

| | | | | | | | |
|-----|----|-----|----|----|----|--|--|
| | | A B | | | | | |
| | | 00 | 01 | 11 | 10 | | |
| C D | 00 | 0 | 4 | 12 | 8 | | |
| | 01 | 1 | 5 | 13 | 9 | | |
| | 11 | 3 | 7 | 15 | 11 | | |
| | 10 | 2 | 6 | 14 | 10 | | |
| | | 0 | 4 | 12 | 8 | | |
| | | 1 | 5 | 13 | 9 | | |
| | | 3 | 7 | 15 | 11 | | |
| | | 2 | 6 | 14 | 10 | | |
| | | 0 | 4 | 12 | 8 | | |
| | | 1 | 5 | 13 | 9 | | |
| | | 3 | 7 | 15 | 11 | | |
| | | 2 | 6 | 14 | 10 | | |

| | | | | | | | |
|-----|----|-----|----|----|----|--|--|
| | | A B | | | | | |
| | | 00 | 01 | 11 | 10 | | |
| C D | 00 | 0 | 4 | 12 | 8 | | |
| | 01 | 1 | 5 | 13 | 9 | | |
| | 11 | 3 | 7 | 15 | 11 | | |
| | 10 | 2 | 6 | 14 | 10 | | |
| | | 0 | 4 | 12 | 8 | | |
| | | 1 | 5 | 13 | 9 | | |
| | | 3 | 7 | 15 | 11 | | |
| | | 2 | 6 | 14 | 10 | | |
| | | 0 | 4 | 12 | 8 | | |
| | | 1 | 5 | 13 | 9 | | |
| | | 3 | 7 | 15 | 11 | | |
| | | 2 | 6 | 14 | 10 | | |

$$\text{MSP1} = x\bar{z} + x*y + w\bar{y}*z + \bar{w}\bar{x}\bar{y}$$

$$\text{MSP2} = x*y + w*x + \bar{x}\bar{y}*z + \bar{w}\bar{y}\bar{z}$$

2.

| | | | | | | | |
|-----|----|-----|----|----|----|--|--|
| | | A B | | | | | |
| | | 00 | 01 | 11 | 10 | | |
| C D | 00 | 0 | 4 | 12 | 8 | | |
| | 01 | 1 | 5 | 13 | 9 | | |
| | 11 | 3 | 7 | 15 | 11 | | |
| | 10 | 2 | 6 | 14 | 10 | | |
| | | 0 | 4 | 12 | 8 | | |
| | | 1 | 5 | 13 | 9 | | |
| | | 3 | 7 | 15 | 11 | | |
| | | 2 | 6 | 14 | 10 | | |
| | | 0 | 4 | 12 | 8 | | |
| | | 1 | 5 | 13 | 9 | | |
| | | 3 | 7 | 15 | 11 | | |
| | | 2 | 6 | 14 | 10 | | |

| | | | | | | | |
|-----|----|-----|----|----|----|--|--|
| | | A B | | | | | |
| | | 00 | 01 | 11 | 10 | | |
| C D | 00 | 0 | 4 | 12 | 8 | | |
| | 01 | 1 | 5 | 13 | 9 | | |
| | 11 | 3 | 7 | 15 | 11 | | |
| | 10 | 2 | 6 | 14 | 10 | | |
| | | 0 | 4 | 12 | 8 | | |
| | | 1 | 5 | 13 | 9 | | |
| | | 3 | 7 | 15 | 11 | | |
| | | 2 | 6 | 14 | 10 | | |
| | | 0 | 4 | 12 | 8 | | |
| | | 1 | 5 | 13 | 9 | | |
| | | 3 | 7 | 15 | 11 | | |
| | | 2 | 6 | 14 | 10 | | |

$$\text{MPS1} = (x + \bar{y})(w + \bar{x} + y + \bar{z})(\bar{w} + x + z)$$

$$\text{MPS2} = (x + \bar{y})(w + \bar{x} + y + \bar{z})(\bar{w} + y + z)$$

3. Neither the MSP nor the MPS are unique.

4. $\text{MPS1} = \text{MSP1} = \text{MSP2}$

Problem 4

1. $\overline{(\overline{xy}z)} = x + \overline{y} + z$

2. $\overline{(a + \overline{b})(c + \overline{d})} = \overline{(a + \overline{b})} + \overline{(c + \overline{d})} = \overline{a} * b + \overline{c} * d$

3. $\overline{(\overline{a + b + c})(b + c)} = \overline{a} * \overline{b} * \overline{c} * b * c = \overline{a} * \overline{b} * \overline{c}$

Problem 5

```
-----  
-- 6.111 Problem Set 1: Problem 5  
library ieee;  
use ieee.std_logic_1164.all;  
  
-- Entity  
entity prob5 is port (a, b, c, d : in std_logic;  
                    p1, p2 : out std_logic);  
end prob5;  
  
-- Architecture  
architecture comp of prob5 is  
begin  
    p1 <= not(a and b and (not c));  
    p2 <= not((not(c) or not(d)) and (a or not(b)));  
end comp;  
-----
```

Contents of p5.vhd

```
-----  
DESIGN EQUATIONS      (15:33:57)
```

$$/p1 = a * b * /c$$

$$p2 = c * d + /a * b$$

Completed Successfully

```
-----  
Taken from p5.rpt
```

Problem 6

- 2to1 Multiplexer

```
library ieee;
use ieee.std_logic_1164.all;

-- Entity
entity mux21 is
    generic (length: integer := 4);
    port (a, b : in std_logic_vector((length - 1) downto 0);
          s : in std_logic;
          x : out std_logic_vector((length - 1) downto 0));
end mux21;

-- Architecture
architecture comp of mux21 is
begin
    with s select
        x <= a when '0',
            b when others; -- Includes '1', but catches all others
end comp;
```

```
-----  
-- 4to1 Multiplexer  
-----
```

```
library ieee;
```

```
use ieee.std_logic_1164.all;
```

```
-- Entity
```

```
entity mux41 is
```

```
    generic (length: integer := 4);
```

```
    port (a, b, c, d :in std_logic_vector((length - 1) downto 0);
```

```
          s : in std_logic_vector(1 downto 0);
```

```
          x : out std_logic_vector((length - 1) downto 0));
```

```
end mux41;
```

```
-- Architecture
```

```
architecture comp of mux41 is
```

```
begin
```

```
    with s select
```

```
        x <= a when '00',
```

```
            b when '01',
```

```
            c when '10',
```

```
            d when others; --Includes '11', but catches all others
```

```
end comp;
```